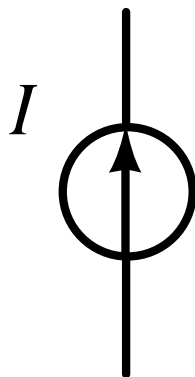


Strujni izvori (Izvori konstantne struje)

- Za razliku od naponskih izvora **izvor konstantne struje** u grani kola u kojoj se nalazi daju **svoju nazivnu struju** bez obzira na **vrijednost otpornosti** u toj grani
- **Smijer struje** koju daje **strujni izvor** naznačen je **strelicom** u njegovom simbolu na šemi i taj smjer struje predstavlja istovremeno i **smjer struje u toj grani kola**
- Napon na krajevima strujnog izvora zavisi od vrijednosti otpornosti u grani koka gdje se strujni izvor nalazi
- **Idealni** strujni izvor ima **beskonačnu unutrašnju otpornost**



Idealni strujni izvor

Strujni izvori (Izvori konstantne struje)

Primjer:

 **EXAMPLE 8-1** Refer to the circuit of Figure 8-3:

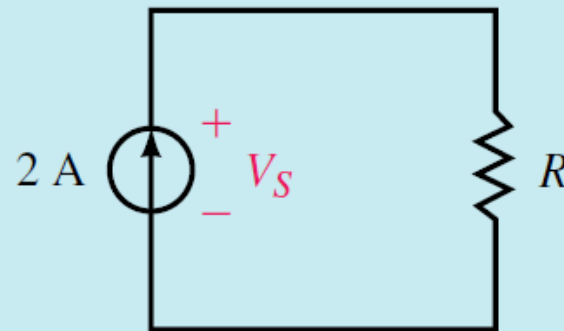


FIGURE 8-3

- Calculate the voltage V_S across the current source if the resistor is $100\ \Omega$.
- Calculate the voltage if the resistor is $2\ \text{k}\Omega$.

Solution The current source maintains a constant current of $2\ \text{A}$ through the circuit. Therefore,

- $V_S = V_R = (2\ \text{A})(100\ \Omega) = 200\ \text{V}.$
- $V_S = V_R = (2\ \text{A})(2\ \text{k}\Omega) = 4000\ \text{V}.$

Strujni izvori (Izvori konstantne struje)

Primjer:

EXAMPLE 8-2 Determine the voltages V_1 , V_2 , and V_S and the current I_S for the circuit of Figure 8-4.

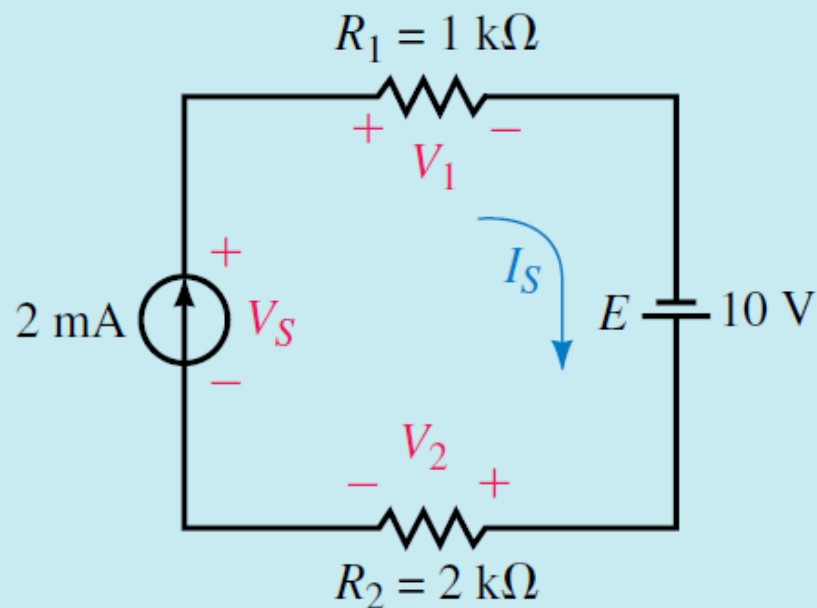


FIGURE 8-4

Strujni izvori (Izvori konstantne struje)

Primjer:

Solution Since the given circuit is a series circuit, the current everywhere in the circuit must be the same, namely

$$I_s = 2 \text{ mA}$$

Using Ohm's law,

$$V_1 = (2 \text{ mA})(1 \text{ k}\Omega) = 2.00 \text{ V}$$

$$V_2 = (2 \text{ mA})(2 \text{ k}\Omega) = 4.00 \text{ V}$$

Applying Kirchhoff's voltage law around the closed loop,

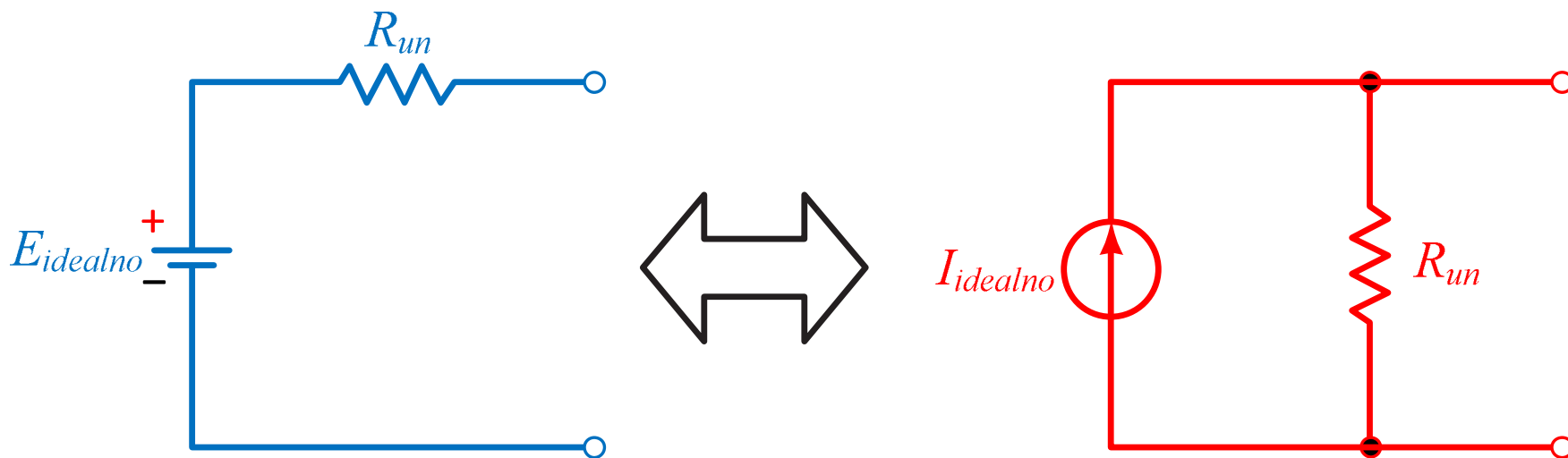
$$\sum V = V_s - V_1 - V_2 + E = 0$$

$$\begin{aligned} V_s &= V_1 + V_2 - E \\ &= 2 \text{ V} + 4 \text{ V} - 10 \text{ V} = -4.00 \text{ V} \end{aligned}$$

From the above result, you see that the actual polarity of V_s is opposite to that assumed.

Konverzija između naponskih i strujnih izvora

- Realni naponski izvor ima malu unutrašnju otpornost R_{un} koja je povezana u seriju sa idealnim naponskim izvorom
- Realni strujni izvor ima veliku unutrašnju otpornost R_{un} koja je vezana paralelno sa idealnim strujnim izvorom
- Moguća je konverzija realnog naponskog u realni strujni izvor i obrnuto

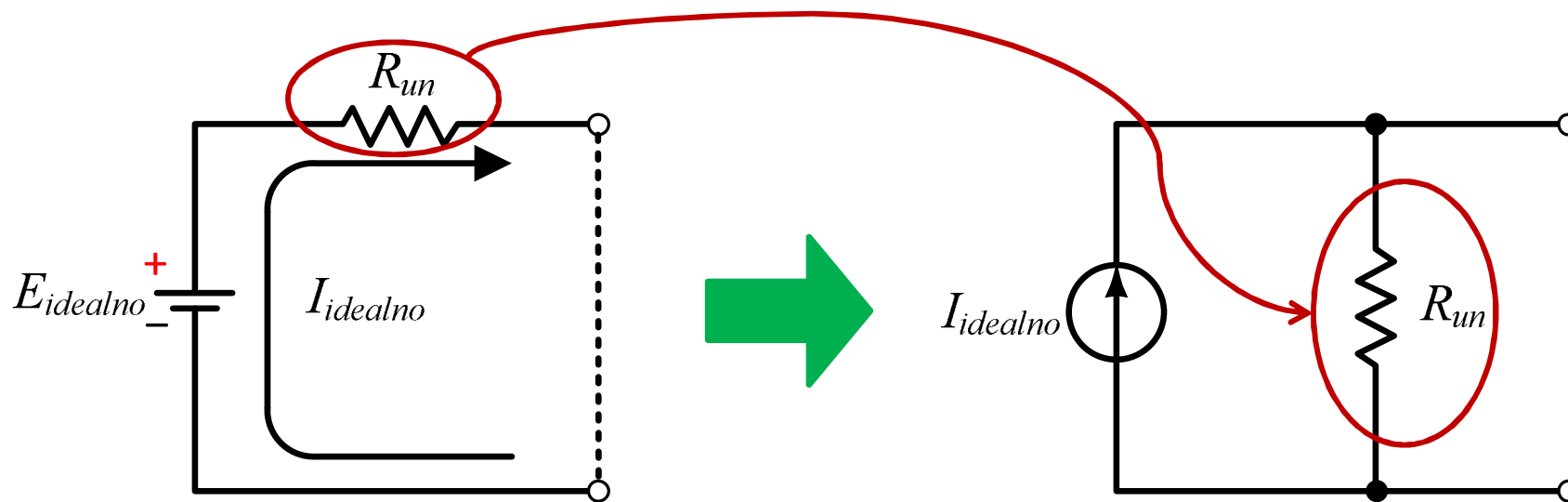


Konverzija realnog naponskog u realni strujni izvor

- Pretpostavimo da su priključci realnog naponskog izvora kratko spojeni. Tada je struja u kolu:

$$I_{\text{idealno}} = \frac{E_{\text{idealno}}}{R_{un}}$$

- To je struja ekvivalentnog realnog strujnog izvora

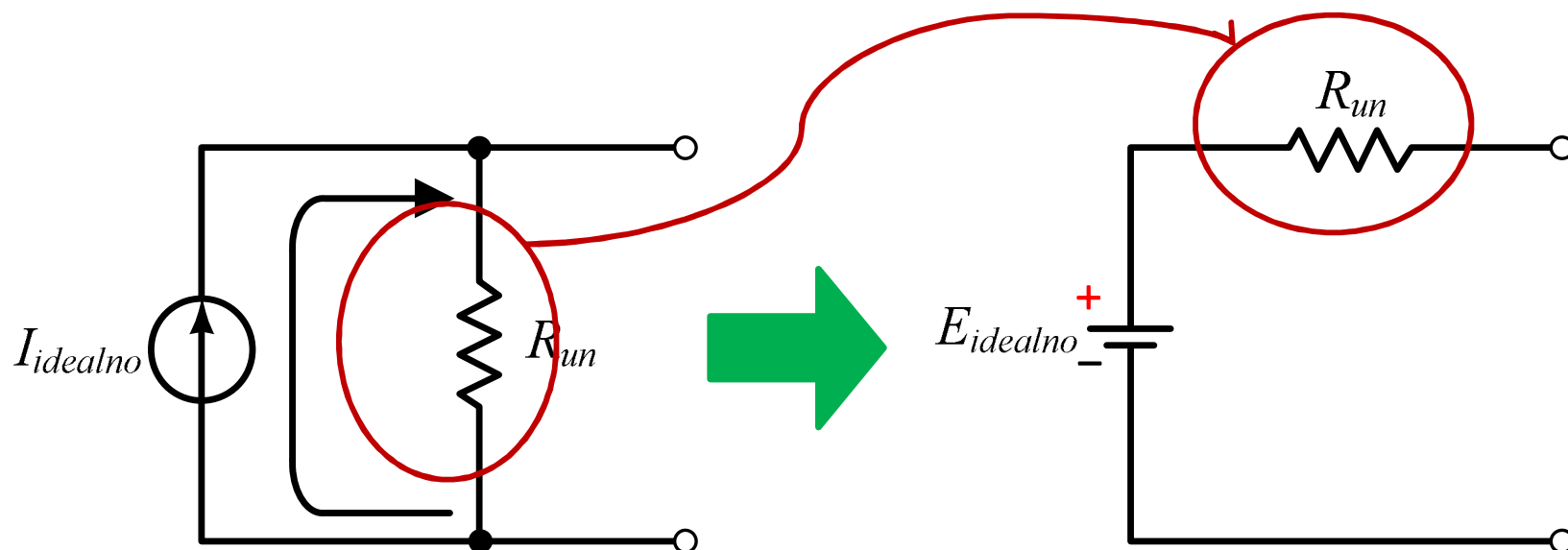


Konverzija realnog strujnog u realni realni izvor

- Pretpostavimo da su **priključci realnog strujnog izvora otvoreni**. Tada je napon na krajevima strunog izvora:

$$E_{\text{idealno}} = I_{\text{idealno}} \cdot R_{un}$$

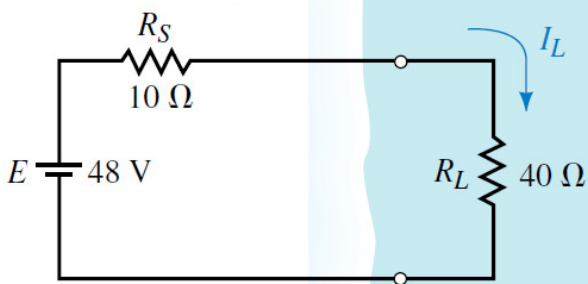
- To je napon ekvivalentnog realnog naponskog izvora



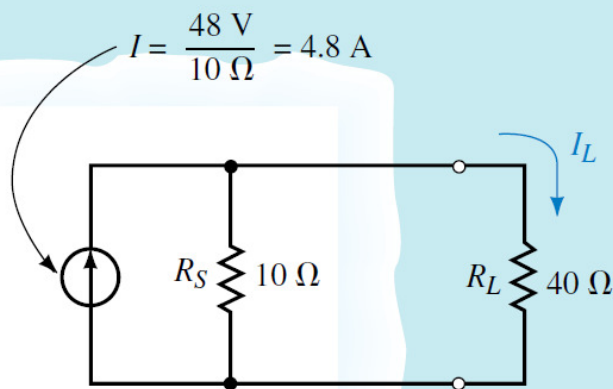
Konverzija između naponskih i strujnih izvora

Primjer:

EXAMPLE 8-4 Convert the voltage source of Figure 8-9(a) into a current source and verify that the current, I_L , through the load is the same for each source.



(a)



Konverzija između naponskih i strujnih izvora

Primjer:

Solution The equivalent current source will have a current magnitude given as

$$I = \frac{48 \text{ V}}{10 \Omega} = 4.8 \text{ A}$$

The resulting circuit is shown in Figure 8–9(b).

For the circuit of Figure 8–9(a), the current through the load is found as

$$I_L = \frac{48 \text{ V}}{10 \Omega + 40 \Omega} = 0.96 \text{ A}$$

For the equivalent circuit of Figure 8–9(b), the current through the load is

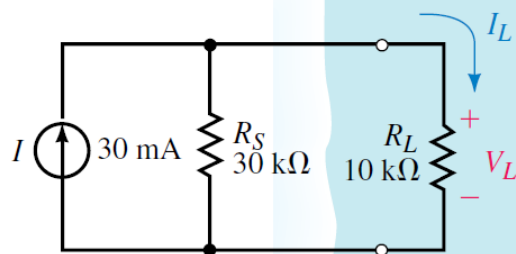
$$I_L = \frac{(4.8 \text{ A})(10 \Omega)}{10 \Omega + 40 \Omega} = 0.96 \text{ A}$$

Clearly the results are the same.

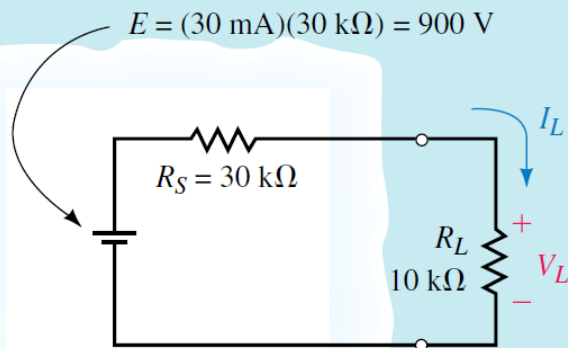
Konverzija između naponskih i strujnih izvora

Primjer:

EXAMPLE 8-5 Convert the current source of Figure 8-10(a) into a voltage source and verify that the voltage, V_L , across the load is the same for each source.



(a)



Konverzija između naponskih i strujnih izvora

Primjer:

Solution The equivalent voltage source will have a magnitude given as

$$E = (30 \text{ mA})(30 \text{ k}\Omega) = 900 \text{ V}$$

The resulting circuit is shown in Figure 8–10(b).

For the circuit of Figure 8–10(a), the voltage across the load is determined as

$$I_L = \frac{(30 \text{ k}\Omega)(30 \text{ mA})}{30 \text{ k}\Omega + 10 \text{ k}\Omega} = 22.5 \text{ mA}$$

$$V_L = I_L R_L = (22.5 \text{ mA})(10 \text{ k}\Omega) = 225 \text{ V}$$

For the equivalent circuit of Figure 8–10(b), the voltage across the load is

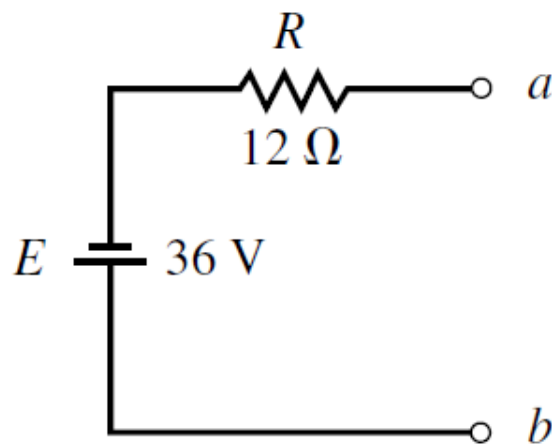
$$V_L = \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 30 \text{ k}\Omega} (900 \text{ V}) = 225 \text{ V}$$

Once again, we see that the circuits are equivalent.

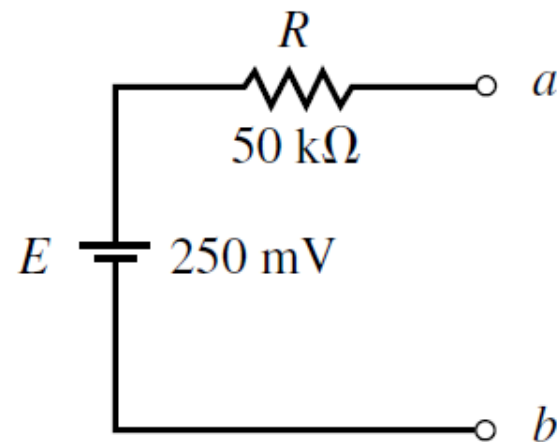
Konverzija između naponskih i strujnih izvora

Primjer:

1. Convert the voltage sources of Figure 8–11 into equivalent current sources.



(a)



(b)

FIGURE 8–11

Konverzija između naponskih i strujnih izvora

Primjer:

2. Convert the current sources of Figure 8–12 into equivalent voltage sources.

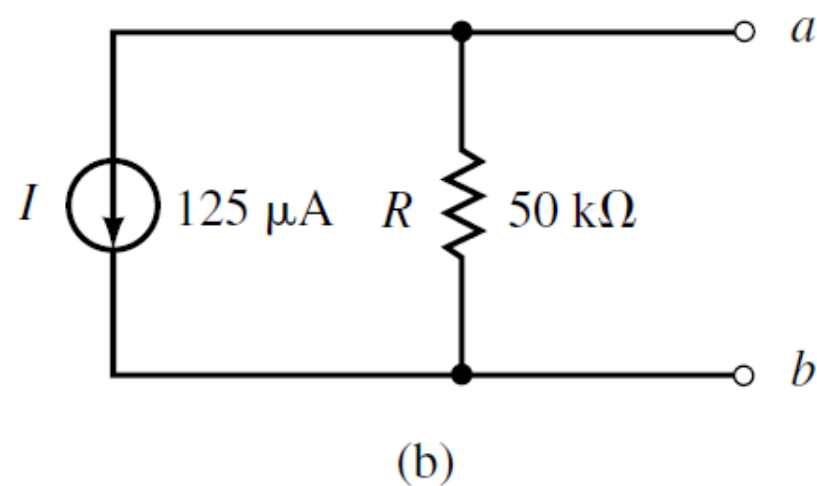
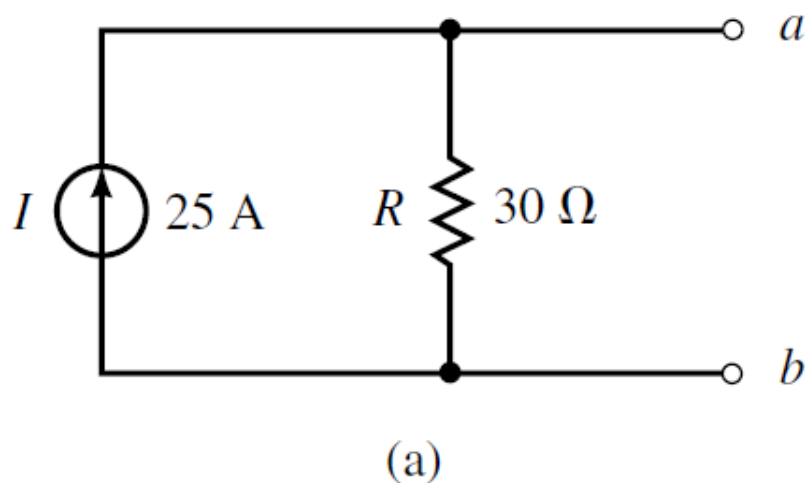


FIGURE 8–12

Direktna primjena Kirhofovih zakona za analizu kola

- Osnovni zadatak u primjeni ove metode svodi se na postavljanje dovoljnog broja jednačina na bazi I i II Kirhofovog zakona
- Pri primjeni ove metode potrebno se pridržavati sljedećih pravila:
 1. **Označiti smijer struje** u svakoj grani kola. Ako grana kola sadrži strujni izvor onda preskočiti ovaj korak
 2. Za usvojeni smijer struje **označiti padove napona** na svim otpornicima u kolu
 3. **Primjeniti II Kirhofov zakon** za svaku zatvorenu strujnu konturu. Broj jednačina je $N_G - (N_{\check{c}} - 1)$. Ako se u grani nalazi samo idealni strujni izvor, tu granu nije potrebno uključivati sa ostalim strujnim konturama
 4. **Primjeniti I Kirhofov zakon** za svaki čvor u kolu. Maksimalno se može postaviti $N_{\check{c}} - 1$ jednačina

Direktna primjena Kirhofovih zakona za analizu kola

Primjer:

EXAMPLE 8-8 Find the current in each branch in the circuit of Figure 8-18.

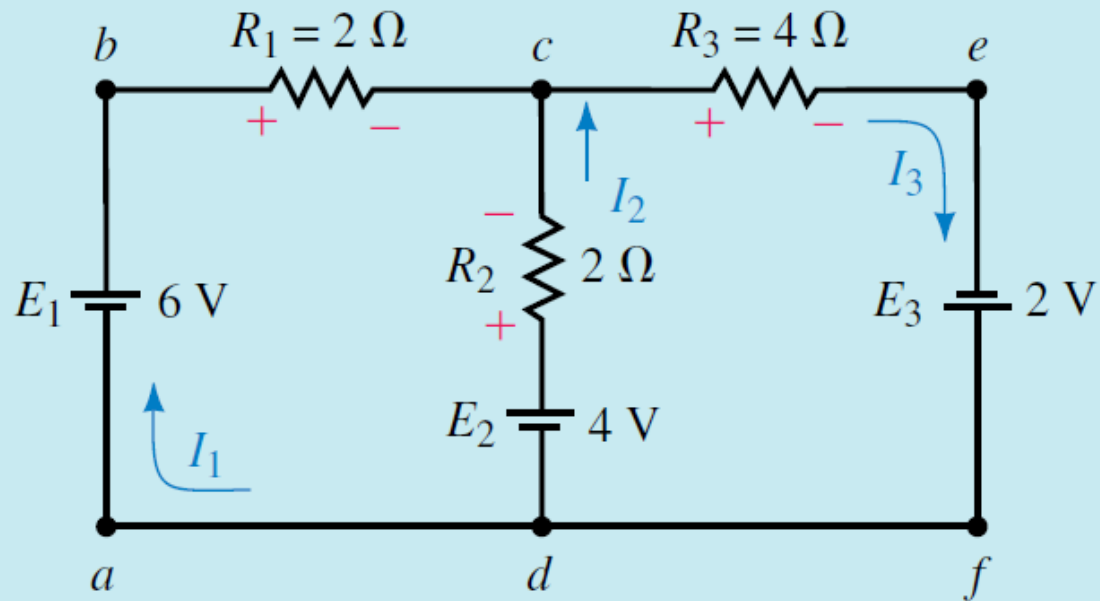


FIGURE 8-18

Direktna primjena Kirhofovih zakona za analizu kola

Primjer:

Solution

Step 1: Assign currents as shown in Figure 8–18.

Step 2: Indicate the polarities of the voltage drops on all resistors in the circuit, using the assumed current directions.

Step 3: Write the Kirchhoff voltage law equations.

Loop *abcd*a: $6\text{ V} - (2\ \Omega)I_1 + (2\ \Omega)I_2 - 4\text{ V} = 0\text{ V}$

Notice that the circuit still has one branch which has not been included in the KVL equations, namely the branch *cefd*. This branch would be included if a loop equation for *cefdc* or for *abcefd*a were written. There is no reason for choosing one loop over another, since the overall result will remain unchanged even though the intermediate steps will not give the same results.

Direktna primjena Kirhofovih zakona za analizu kola

Primjer:

$$\text{Loop } cefdc: \quad 4 \text{ V} - (2 \, \Omega)I_2 - (4 \, \Omega)I_3 + 2 \text{ V} = 0 \text{ V}$$

Now that all branches have been included in the loop equations, there is no need to write any more. Although more loops exist, writing more loop equations would needlessly complicate the calculations.

Step 4: Write the Kirchhoff current law equation(s).

By applying KCL at node c , all branch currents in the network are included.

$$\text{Node } c: \quad I_3 = I_1 + I_2$$

To simplify the solution of the simultaneous linear equations we write them as follows:

$$2I_1 - 2I_2 + 0I_3 = 2$$

$$0I_1 - 2I_2 - 4I_3 = -6$$

$$1I_1 + 1I_2 - 1I_3 = 0$$

Direktna primjena Kirhofovih zakona za analizu kola

Primjer:

The principles of linear algebra (Appendix B) allow us to solve for the determinant of the denominator as follows:

$$\begin{aligned} D &= \begin{vmatrix} 2 & -2 & 0 \\ 0 & -2 & -4 \\ 1 & 1 & -1 \end{vmatrix} \\ &= 2 \begin{vmatrix} -2 & -4 \\ 1 & -1 \end{vmatrix} - 0 \begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix} + 1 \begin{vmatrix} -2 & 0 \\ -2 & -4 \end{vmatrix} \\ &= 2(2 + 4) - 0 + 1(8) = 20 \end{aligned}$$

Now, solving for the currents, we have the following:

$$\begin{aligned} I_1 &= \frac{\begin{vmatrix} 2 & -2 & 0 \\ -6 & -2 & -4 \\ 0 & 1 & -1 \end{vmatrix}}{D} \\ &= \frac{2 \begin{vmatrix} -2 & -4 \\ 1 & -1 \end{vmatrix} - (-6) \begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix} + 0 \begin{vmatrix} -2 & 0 \\ -2 & -4 \end{vmatrix}}{20} \\ &= \frac{2(2 + 4) + 6(2) + 0}{20} = \frac{24}{20} = 1.200 \text{ A} \end{aligned}$$

Direktna primjena Kirhofovih zakona za analizu kola

Primjer:

$$\begin{aligned} I_2 &= \frac{\begin{vmatrix} 2 & 2 & 0 \\ 0 & -6 & -4 \\ 1 & 0 & -1 \end{vmatrix}}{D} \\ &= \frac{2 \begin{vmatrix} -6 & -4 \\ 0 & -1 \end{vmatrix} - 0 \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 0 \\ -6 & -4 \end{vmatrix}}{20} \\ &= \frac{2(6) + 0 + 1(-8)}{20} = \frac{4}{20} = 0.200 \text{ A} \end{aligned}$$
$$\begin{aligned} I_3 &= \frac{\begin{vmatrix} 2 & -2 & 2 \\ 0 & -2 & -6 \\ 1 & 1 & 0 \end{vmatrix}}{D} \\ &= \frac{2 \begin{vmatrix} -2 & -6 \\ 1 & 0 \end{vmatrix} - 0 \begin{vmatrix} -2 & 2 \\ 1 & 0 \end{vmatrix} + 1 \begin{vmatrix} -2 & 2 \\ -2 & -6 \end{vmatrix}}{20} \\ &= \frac{2(6) - 0 + 1(12 + 4)}{20} = \frac{28}{20} = 1.400 \text{ A} \end{aligned}$$

Direktna primjena Kirhofovih zakona za analizu kola

Primjer:

EXAMPLE 8–9 Find the currents in each branch of the circuit shown in Figure 8–19. Solve for the voltage V_{ab} .

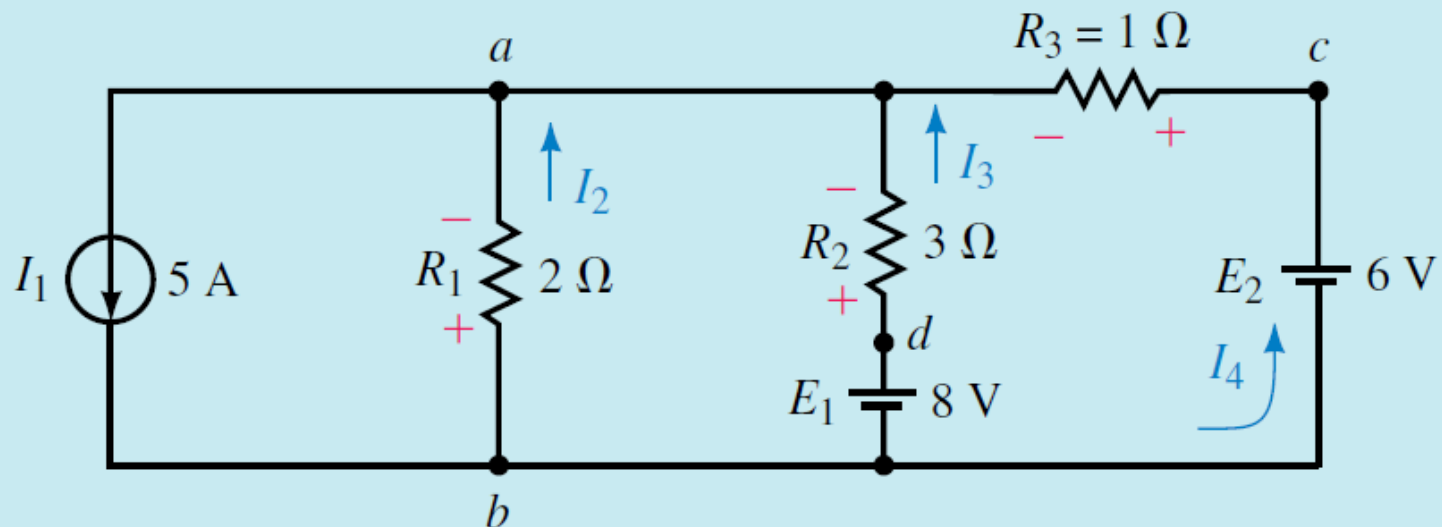


FIGURE 8–19

Direktna primjena Kirhofovih zakona za analizu kola

Primjer:

Solution Notice that although the above circuit has four currents, there are only three **unknown** currents: I_2 , I_3 , and I_4 . The current I_1 is given by the value of the constant-current source. In order to solve this network we will need three linear equations. As before, the equations are determined by Kirchhoff's voltage and current laws.

Step 1: The currents are indicated in the given circuit.

Step 2: The polarities of the voltages across all resistors are shown.

Step 3: Kirchhoff's voltage law is applied at the indicated loops:

$$\text{Loop } badb: \quad -(2\ \Omega)(I_2) + (3\ \Omega)(I_3) - 8\ \text{V} = 0\ \text{V}$$

$$\text{Loop } bacb: \quad -(2\ \Omega)(I_2) + (1\ \Omega)(I_4) - 6\ \text{V} = 0\ \text{V}$$

Step 4: Kirchhoff's current law is applied as follows:

$$\text{Node } a: \quad I_2 + I_3 + I_4 = 5\ \text{A}$$

Direktna primjena Kirhofovih zakona za analizu kola

Primjer: Rewriting the linear equations,

$$-2I_2 + 3I_3 + 0I_4 = 8$$

$$-2I_2 + 0I_3 + 1I_4 = 6$$

$$1I_2 + 1I_3 + 1I_4 = 5$$

The determinant of the denominator is evaluated as

$$D = \begin{vmatrix} -2 & 3 & 0 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 11$$

Now solving for the currents, we have

$$I_2 = \frac{\begin{vmatrix} 8 & 3 & 0 \\ 6 & 0 & 1 \\ 5 & 1 & 1 \end{vmatrix}}{D} = \frac{11}{11} = -1.00 \text{ A}$$

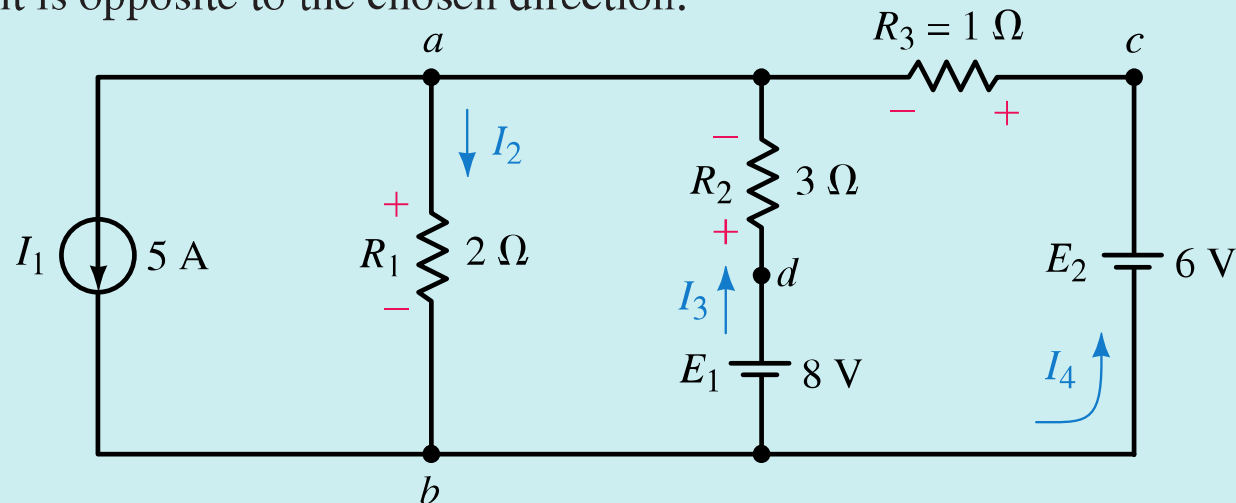
$$I_3 = \frac{\begin{vmatrix} -2 & 8 & 0 \\ -2 & 6 & 1 \\ 1 & 5 & 1 \end{vmatrix}}{D} = \frac{22}{11} = 2.00 \text{ A}$$

Direktna primjena Kirhofovih zakona za analizu kola

Primjer:

$$I_4 = \frac{\begin{vmatrix} -2 & 3 & 8 \\ -2 & 0 & 6 \\ 1 & 1 & 5 \end{vmatrix}}{D} = \frac{44}{11} = 4.00 \text{ A}$$

The current I_2 is negative, which simply means that the actual direction of the current is opposite to the chosen direction.



$$I_2 = 1.00 \text{ A}$$

$$I_3 = 2.00 \text{ A}$$

$$I_4 = 4.00 \text{ A}$$

Using the actual direction for I_2 ,

$$V_{ab} = +(2\ \Omega)(1\text{ A}) = +2.00 \text{ V}$$

Metoda konturnih struja

- Metoda konturnih struja počiva na primjeni II Kirhofovog zakona, sa manjim brojem jednačina
- Broj jednačina koje treba postaviti je $N_G - (N_{\check{C}} - 1)$
- Pri primjeni ove metode potrebno se pridržavati sljedećih pravila:
 1. Pretvoriti sve strujne izvore u ekvivalentne naponske izvore
 2. Za svaku nezavisnu konturu proizvoljno usvojiti smijer glavne konturne struje
 3. Za usvojene smijer glavne konturne struje označiti padove napona na svakom otporniku u konturi. Za otpornike u zajedničkim granama označiti padove napona koje prave sve glavne konturne struje
 4. Primjeniti II Kirhofov zakon za sve nezavisne konture
 5. Struje u zajedničkim granama odrediti kao algebarsku suma/razlika glavnih konturnih struja

Metoda konturnih struja

- Metoda konturnih struja počiva na **primjeni II Kirhofovog zakona**, sa manjim brojem jednačina
- Broj jednačina koje treba postaviti je **$N_G - (N_{\check{c}} - 1)$**
- Pri primjeni ove metode potrebno se pridržavati sljedećih pravila:
 1. Pretvoriti sve strujne izvore u ekvivalentne naponske izvore
 2. Za svaku nezavisnu konturu proizvoljno usvojiti smijer glavne konturne struje
 3. Za usvojene smijer glavne konturne struje označiti padove napona na svakom otporniku u konturi. Za otpornike u zajedničkim granama označiti padove napona koje prave sve glavne konturne struje
 4. Primjeniti II Kirhofov zakon za sve nezavisne konture
 5. Struje u zajedničkim granama odrediti kao algebarsku suma/razlika glavnih konturnih struja

Metoda konturnih struja

Primjer:

EXAMPLE 8-10 Find the current in each branch for the circuit of Figure 8-22.

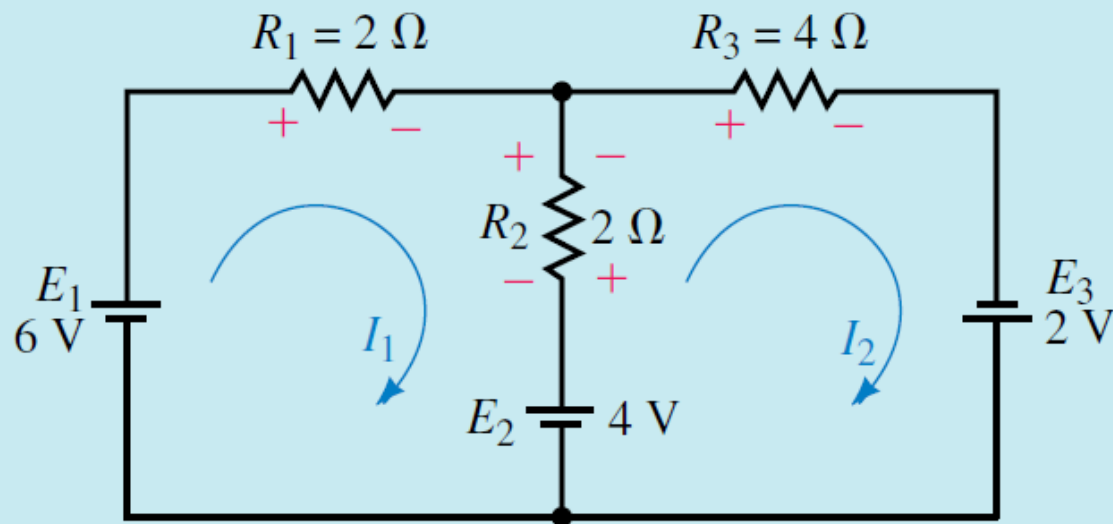


FIGURE 8-22

Solution

Step 1: Loop currents are assigned as shown in Figure 8-22. These currents are designated I_1 and I_2 .

Metoda konturnih struja

Primjer:

Step 2: Voltage polarities are assigned according to the loop currents. Notice that the resistor R_2 has two different voltage polarities due to the different loop currents.

Step 3: The loop equations are written by applying Kirchhoff's voltage law in each of the loops. The equations are as follows:

$$\text{Loop 1:} \quad 6 \text{ V} - (2 \, \Omega)I_1 - (2 \, \Omega)I_1 + (2 \, \Omega)I_2 - 4 \text{ V} = 0$$

$$\text{Loop 2:} \quad 4 \text{ V} - (2 \, \Omega)I_2 + (2 \, \Omega)I_1 - (4 \, \Omega)I_2 + 2 \text{ V} = 0$$

Note that the voltage across R_2 due to the currents I_1 and I_2 is indicated as two separated terms, where one term represents a voltage drop in the direction of I_1 and the other term represents a voltage rise in the same direction. The magnitude and polarity of the voltage across R_2 is determined by the actual size and directions of the loop currents. The above loop equations may be simplified as follows:

$$\text{Loop 1:} \quad (4 \, \Omega)I_1 - (2 \, \Omega)I_2 = 2 \text{ V}$$

$$\text{Loop 2:} \quad -(2 \, \Omega)I_1 + (6 \, \Omega)I_2 = 6 \text{ V}$$

Using determinants, the loop equations are easily solved as

$$I_1 = \frac{\begin{vmatrix} 2 & -2 \\ 6 & 6 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ -2 & 6 \end{vmatrix}} = \frac{12 + 12}{24 - 4} = \frac{24}{20} = 1.20 \text{ A}$$

Metoda konturnih struja

Primjer:

and

$$I_2 = \frac{\begin{vmatrix} 4 & 2 \\ -2 & 6 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ -2 & 6 \end{vmatrix}} = \frac{24 + 4}{24 - 4} = \frac{28}{20} = 1.40 \text{ A}$$

From the above results, we see that the currents through resistors R_1 and R_3 are I_1 and I_2 respectively.

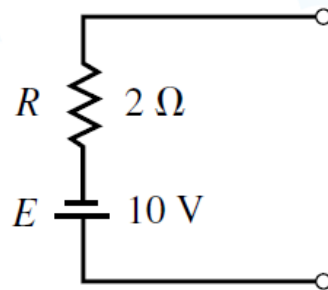
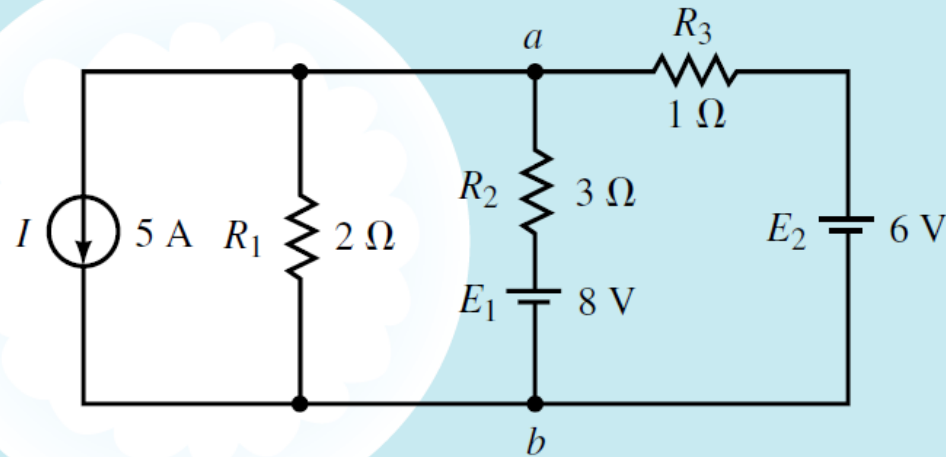
The branch current for R_2 is found by combining the loop currents through this resistor:

$$I_{R_2} = 1.40 \text{ A} - 1.20 \text{ A} = 0.20 \text{ A} \quad (\text{upward})$$

The results obtained by using mesh analysis are exactly the same as those obtained by branch-current analysis. Whereas branch-current analysis required three equations, this approach requires the solution of only two simultaneous linear equations. Mesh analysis also requires that only Kirchhoff's voltage law be applied and clearly illustrates why mesh analysis is preferred to branch-current analysis.

Metoda konturnih struja

Primjer: **EXAMPLE 8-11** Determine the current through the 8-V battery for the circuit shown in Figure 8-23.



Metoda konturnih struja

Primjer:

Solution Convert the current source into an equivalent voltage source. The equivalent circuit may now be analyzed by using the loop currents shown in Figure 8–24.

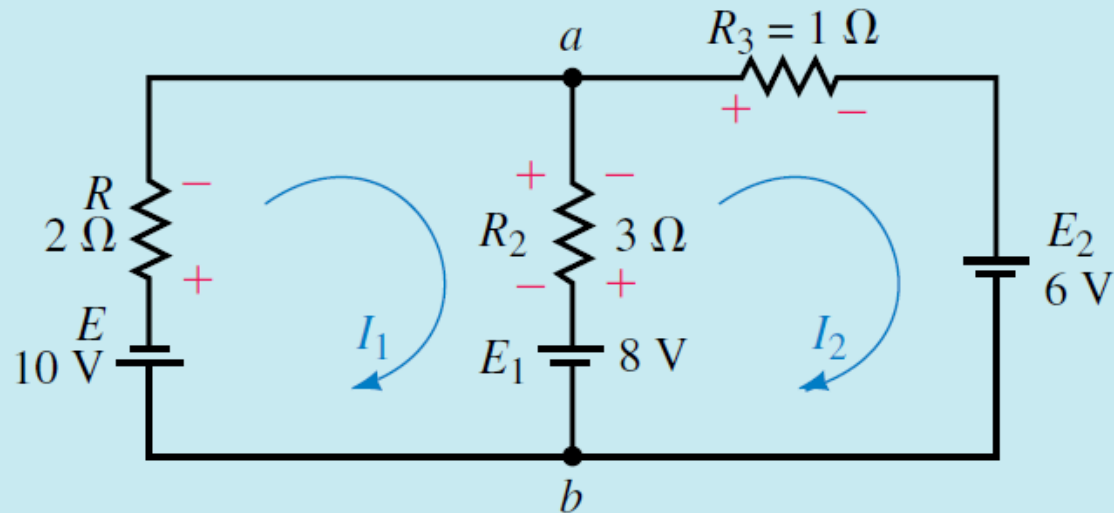


FIGURE 8–24

$$\text{Loop 1:} \quad -10\text{ V} - (2\ \Omega)I_1 - (3\ \Omega)I_1 + (3\ \Omega)I_2 - 8\text{ V} = 0$$

$$\text{Loop 2:} \quad 8\text{ V} - (3\ \Omega)I_2 + (3\ \Omega)I_1 - (1\ \Omega)I_2 - 6\text{ V} = 0$$

Metoda konturnih struja

Primjer:

Rewriting the linear equations, you get the following:

$$\text{Loop 1:} \quad (5 \, \Omega)I_1 - (3 \, \Omega)I_2 = -18 \, \text{V}$$

$$\text{Loop 2:} \quad -(3 \, \Omega)I_1 + (4 \, \Omega)I_2 = 2 \, \text{V}$$

Solving the equations using determinants, we have the following:

$$I_1 = \frac{\begin{vmatrix} -18 & -3 \\ 2 & 4 \end{vmatrix}}{\begin{vmatrix} 5 & -3 \\ -3 & 4 \end{vmatrix}} = -\frac{66}{11} = -6.00 \, \text{A}$$

$$I_2 = \frac{\begin{vmatrix} 5 & -18 \\ -3 & 2 \end{vmatrix}}{\begin{vmatrix} 5 & -3 \\ -3 & 4 \end{vmatrix}} = -\frac{44}{11} = -4.00 \, \text{A}$$

If the assumed direction of current in the 8-V battery is taken to be I_2 , then

$$I = I_2 - I_1 = -4.00 \, \text{A} - (-6.00 \, \text{A}) = 2.00 \, \text{A}$$

The direction of the resultant current is the same as I_2 (upward).

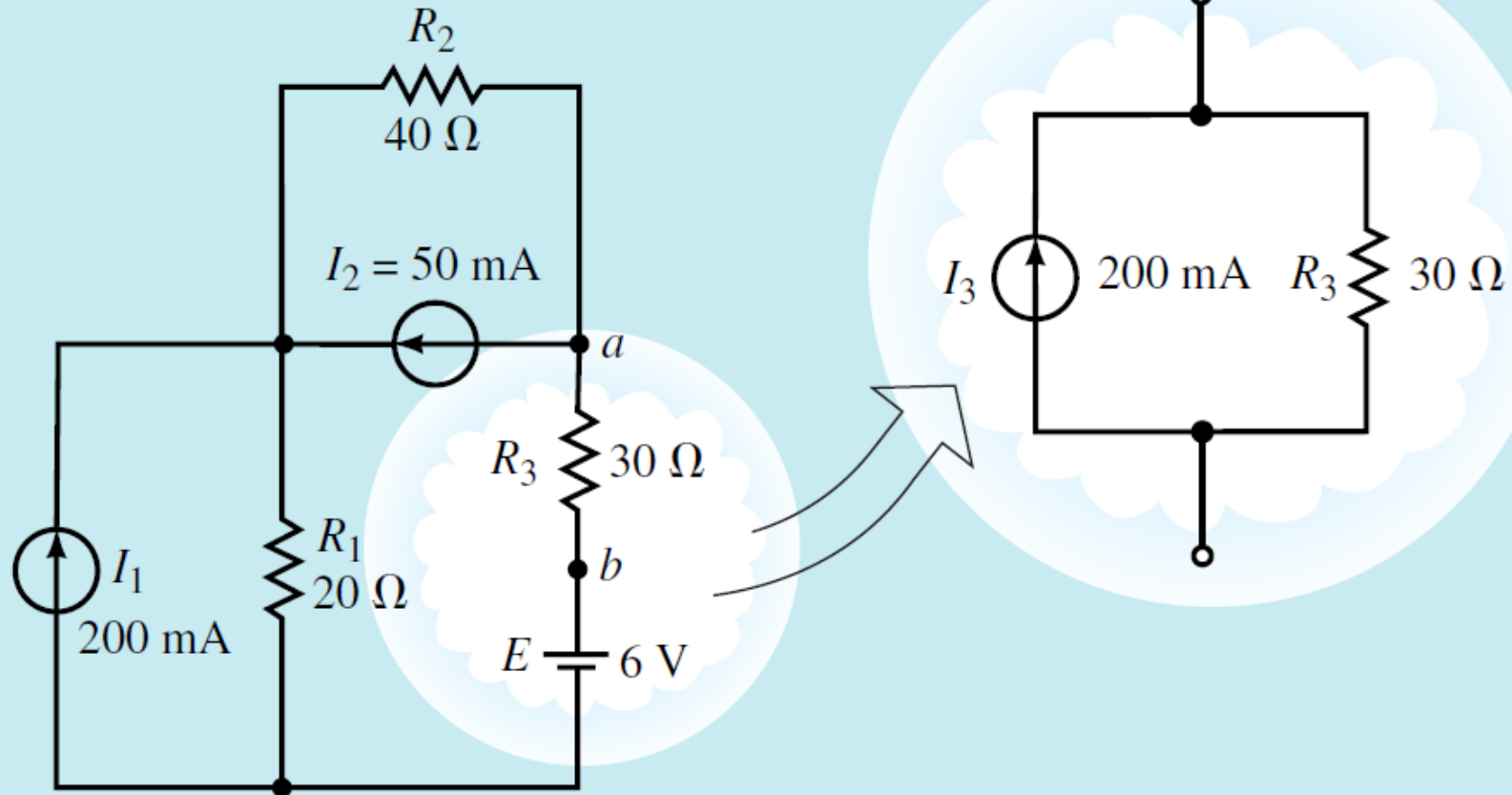
Metoda napona čvorova

- Metoda napona čvorova počiva na **primjeni I Kirhofovog zakona**, sa manjim brojem jednačina
- Pogodna je za složena kola gdje je broj grana znatno veći od broja čvorova. Broj jednačina koje treba postaviti je **$N_{\text{č}}-1$**
- Pri primjeni ove metode potrebno se pridržavati sljedećih pravila:
 1. Proizvoljno izabrati referentni čvor. Ovaj čvor se obično označava kao uzemljenje (masa)
 2. Pretvoriti sve naponske izvore u ekvivalentne strujne izvore
 3. Proizvoljno označiti napone ostalih čvorova u kolu ($U_1, U_2, \dots, U_n$)
 4. Proizvoljno označiti smjerove struja u svakoj nezavisnoj grani gdje nema strujnog izvora. Za izabrane smjerove struja označiti padove napona na otpornicima
 5. Za se čvorove sem referentnog primjeniti I Kirhofovov zakon

Metoda napona čvorova

Primjer:

EXAMPLE 8–14 Given the circuit of Figure 8–30, use nodal analysis to solve for the voltage V_{ab} .



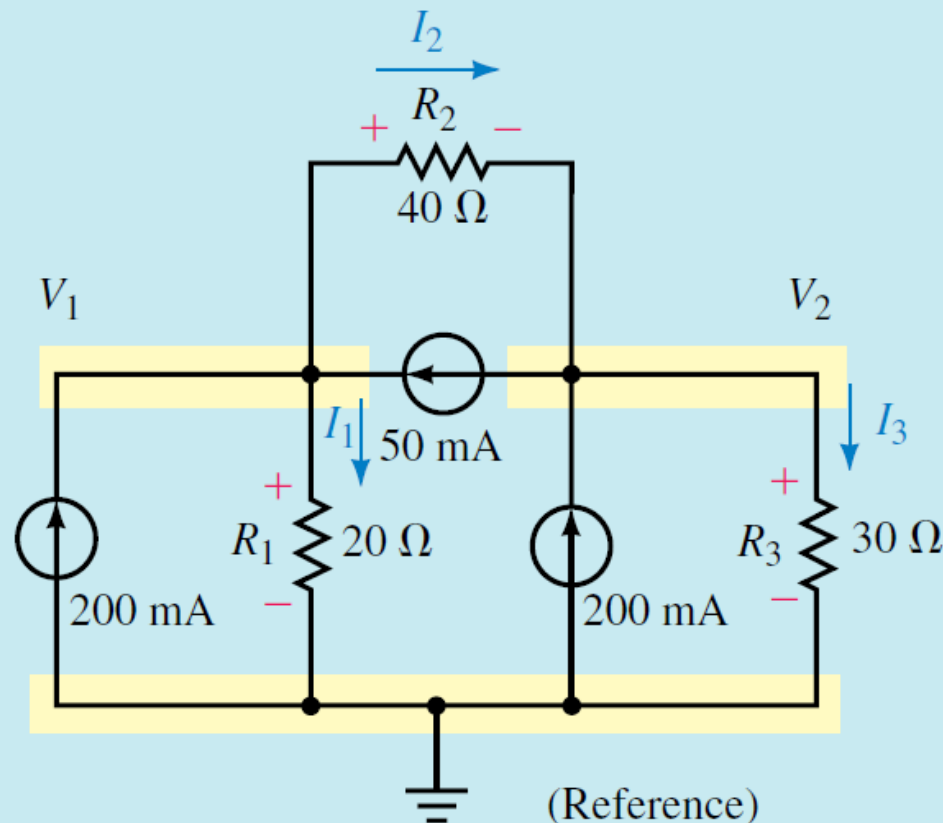
Metoda napona čvorova

Primjer:

Solution

Step 1: Select a convenient reference node.

Step 2: Convert the voltage sources into equivalent current sources. The equivalent circuit is shown in Figure 8–31.



Metoda napona čvorova

Primjer:

Steps 3 and 4: Arbitrarily assign node voltages and branch currents. Indicate the voltage polarities across all resistors according to the assumed current directions.

Step 5: We now apply Kirchhoff's current law at the nodes labelled as V_1 and V_2 :

Node V_1 :

$$\sum I_{\text{entering}} = \sum I_{\text{leaving}}$$
$$200 \text{ mA} + 50 \text{ mA} = I_1 + I_2$$

Node V_2 :

$$\sum I_{\text{entering}} = \sum I_{\text{leaving}}$$
$$200 \text{ mA} + I_2 = 50 \text{ mA} + I_3$$

Step 6: The currents are rewritten in terms of the voltages across the resistors as follows:

$$I_1 = \frac{V_1}{20 \Omega}$$

$$I_2 = \frac{V_1 - V_2}{40 \Omega}$$

$$I_3 = \frac{V_2}{30 \Omega}$$

Metoda napona čvorova

Primjer:

The nodal equations become

$$200 \text{ mA} + 50 \text{ mA} = \frac{V_1}{20 \Omega} + \frac{V_1 - V_2}{40 \Omega}$$

$$200 \text{ mA} + \frac{V_1 - V_2}{40 \Omega} = 50 \text{ mA} + \frac{V_2}{30 \Omega}$$

Substituting the voltage expressions into the original nodal equations, we have the following simultaneous linear equations:

$$\begin{aligned} \left(\frac{1}{20 \Omega} + \frac{1}{40 \Omega} \right) V_1 - \left(\frac{1}{40 \Omega} \right) V_2 &= 250 \text{ mA} \\ - \left(\frac{1}{40 \Omega} \right) V_1 + \left(\frac{1}{30 \Omega} + \frac{1}{40 \Omega} \right) V_2 &= 150 \text{ mA} \end{aligned}$$

These may be further simplified as

$$\begin{aligned} (0.075 \text{ S}) V_1 - (0.025 \text{ S}) V_2 &= 250 \text{ mA} \\ -(0.025 \text{ S}) V_1 + (0.058\bar{3}) V_2 &= 150 \text{ mA} \end{aligned}$$

Metoda napona čvorova

Primjer:

Step 7: Use determinants to solve for the nodal voltages as

$$\begin{aligned} V_1 &= \frac{\begin{vmatrix} 0.250 & -0.025 \\ 0.150 & 0.058\bar{3} \end{vmatrix}}{\begin{vmatrix} 0.075 & -0.025 \\ 0.025 & 0.058\bar{3} \end{vmatrix}} \\ &= \frac{(0.250)(0.058\bar{3}) - (0.150)(-0.025)}{(0.075)(0.058\bar{3}) - (-0.025)(-0.025)} \\ &= \frac{0.018\bar{3}}{0.00375} = 4.89 \text{ V} \end{aligned}$$

Metoda napona čvorova

Primjer:

and

$$\begin{aligned} V_2 &= \frac{\begin{vmatrix} 0.075 & 0.250 \\ -0.025 & 0.150 \end{vmatrix}}{\begin{vmatrix} 0.075 & 0.025 \\ -0.025 & 0.058\bar{3} \end{vmatrix}} \\ &= \frac{(0.075)(0.150) - (-0.025)(0.250)}{0.00375} \\ &= \frac{0.0175}{0.00375} = 4.67 \text{ V} \end{aligned}$$

If we go back to the original circuit of Figure 8–30, we see that the voltage V_2 is the same as the voltage V_a , namely

$$V_a = 4.67 \text{ V} = 6.0 \text{ V} + V_{ab}$$

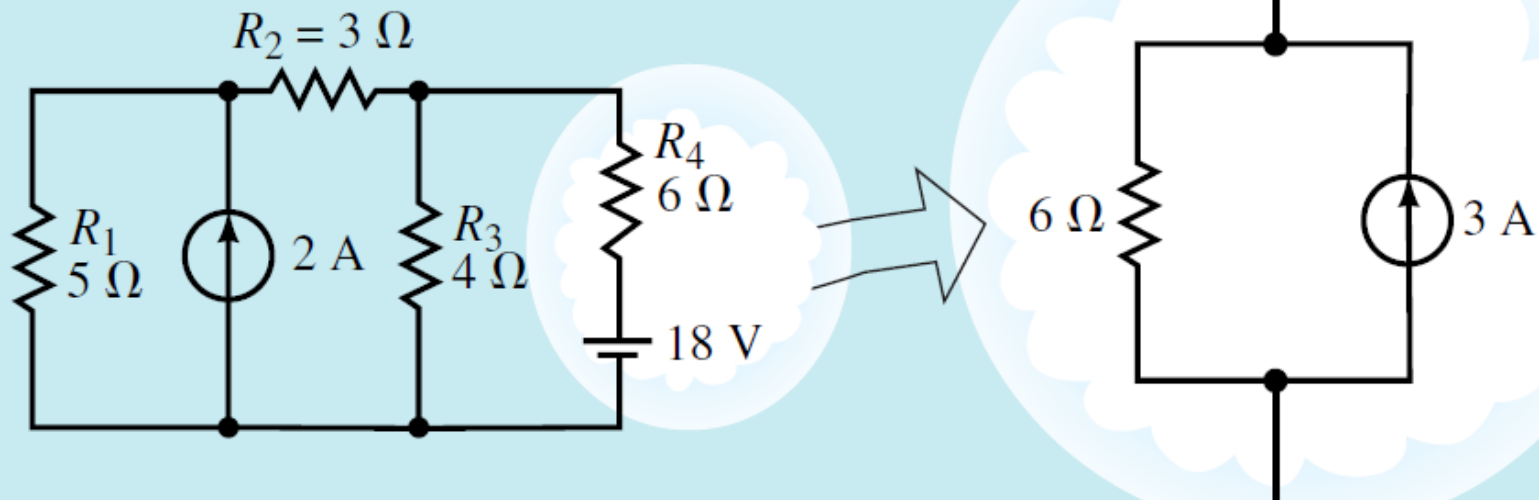
Therefore, the voltage V_{ab} is simply found as

$$V_{ab} = 4.67 \text{ V} - 6.0 \text{ V} = -1.33 \text{ V}$$

Metoda napona čvorova

Primjer:

EXAMPLE 8-15 Determine the nodal voltages for the circuit shown in Figure 8-32.



Metoda napona čvorova

Primjer:

Solution By following the steps outlined, the circuit may be redrawn as shown in Figure 8–33.

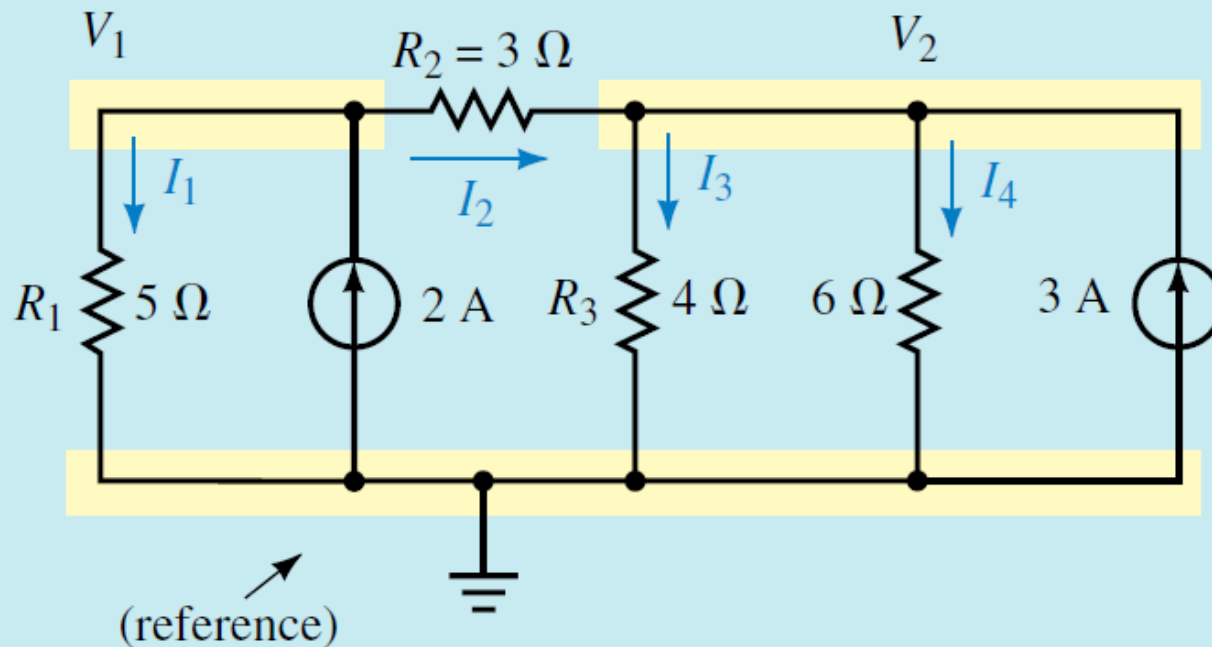


FIGURE 8–33

Metoda napona čvorova

Primjer:

Applying Kirchhoff's current law to the nodes corresponding to V_1 and V_2 , the following nodal equations are obtained:

$$\sum I_{\text{leaving}} = \sum I_{\text{entering}}$$

Node V_1 :

$$I_1 + I_2 = 2 \text{ A}$$

Node V_2 :

$$I_3 + I_4 = I_2 + 3 \text{ A}$$

The currents may once again be written in terms of the voltages across the resistors:

$$I_1 = \frac{V_1}{5 \Omega}$$

$$I_2 = \frac{V_1 - V_2}{3 \Omega}$$

$$I_3 = \frac{V_2}{4 \Omega}$$

$$I_4 = \frac{V_2}{6 \Omega}$$

Metoda napona čvorova

Primjer:

The nodal equations become

$$\text{Node } V_1: \quad \frac{V_1}{5 \, \Omega} + \frac{(V_1 - V_2)}{3 \, \Omega} = 2 \, \text{A}$$

$$\text{Node } V_2: \quad \frac{V_2}{4 \, \Omega} + \frac{V_2}{6 \, \Omega} = \frac{(V_1 - V_2)}{3 \, \Omega} + 3 \, \text{A}$$

These equations may now be simplified as

$$\text{Node } V_1: \quad \left(\frac{1}{5 \, \Omega} + \frac{1}{3 \, \Omega} \right) V_1 - \left(\frac{1}{3 \, \Omega} \right) V_2 = 2 \, \text{A}$$

$$\text{Node } V_2: \quad -\left(\frac{1}{3 \, \Omega} \right) V_1 + \left(\frac{1}{4 \, \Omega} + \frac{1}{6 \, \Omega} + \frac{1}{3 \, \Omega} \right) V_2 = 3 \, \text{A}$$

The solutions for V_1 and V_2 are found using determinants:

$$V_1 = \frac{\begin{vmatrix} 2 & -0.333 \\ 3 & 0.750 \end{vmatrix}}{\begin{vmatrix} 0.533 & -0.333 \\ -0.333 & 0.750 \end{vmatrix}} = \frac{2.500}{0.289} = 8.65 \, \text{V}$$

$$V_2 = \frac{\begin{vmatrix} 0.533 & 2 \\ -0.333 & 3 \end{vmatrix}}{\begin{vmatrix} 0.533 & -0.333 \\ -0.333 & 0.750 \end{vmatrix}} = \frac{2.267}{0.289} = 7.85 \, \text{V}$$