ГРАФИЦИ ФУНКЦИЈА

$\boxed{\text{ЗАД 1:}}$

$$f(x) = \sqrt[3]{x(x-3)^2}$$

Рјешење:

1° Д.П. $\forall x \in R$

2° НУЛЕ: $x(x-3) = 0 \iff x = 0 \vee x = 3$

3° ПАРНОСТ: $f(-x) = \sqrt[3]{-x(-x-3)^2} = -\sqrt[3]{x(x+3)^2}$ није ни парна ни непарна

4° ЗНАК

$f(x) \geq 0$, $x > 0$

$f(x) < 0$, $x < 0$

5° АСИМПТОТЕ

В.А. - нема

коса $k = \lim\limits_{x \to \pm\infty} \frac{f(x)}{x} = \lim\limits_{x \to \pm\infty} \frac{\sqrt[3]{x(x-3)^2}}{x} = \lim\limits_{x \to \pm\infty} \frac{\cancel{x}\sqrt[3]{(1-\frac{3}{x})^2}}{\cancel{x}}$

$\boxed{k = 1}$

$$m = \lim_{x \to \pm\infty} (f(x) - kx) = \lim_{x \to \pm\infty} \left(\sqrt[3]{x(x-3)^2} - x\right) \cdot \frac{\sqrt[3]{x^2(x-3)^4} + x\sqrt[3]{x(x-3)^2} + x^2}{-\|-}$$

$$= \lim_{x \to \pm\infty} \frac{x(x-3)^2 - x^3}{\sqrt[3]{x^2(x-3)^4} + x\sqrt[3]{x(x-3)^2} + x^2} = \lim_{x \to \pm\infty} \frac{-6x^2 + 9x}{x^2\sqrt[3]{(1-\frac{3}{x})^4} + x^2\sqrt[3]{(1-\frac{3}{x})^2} + x^2}$$

$\boxed{m = -2}$ $\qquad$ $\boxed{y = x - 2}$

6° ЕКСТРЕМИ

$$f'(x) = \left[\left(x(x-3)^2\right)^{1/3}\right]' = \frac{1}{3}\left(x(x-3)^2\right)^{-\frac{2}{3}}\left[(x-3)^2 + x \cdot 2(x-3)\right]$$

$$= \frac{x^2 - 6x + 9 + 2x^2 - 6x}{3\sqrt[3]{x^2(x-3)^4}} = \frac{3x^2 - 12x + 9}{3\sqrt[3]{x^2(x-3)^4}} = \frac{x^2 - 4x + 3}{\sqrt[3]{x^2(x-3)^4}} = \frac{(x-1)\cancel{(x-3)}}{\cancel{(x-3)}\sqrt[3]{x^2(x-3)}}$$

$f'(x) = \frac{x-1}{\sqrt[3]{x^2(x-3)}}$, $f'(x)$ - није дефинисана у 0 и 3

$f'(x) = 0 \Rightarrow x = 1$, - стационарна тачка

$\frac{x-1}{x-3}$

	$-\infty$	1		3	$+\infty$
$x-1$	$-$	0	$+$		$+$
$x-3$	$-$		$-$	0	$+$
$f'(x)$	$+$		$-$		$+$
$f(x)$	$\nearrow$	$\frown$	$\searrow$	$\vee$	$\nearrow$

$f(1) = \sqrt[3]{(1-3)^2} = \sqrt[3]{4}$ $(1, \sqrt[3]{4})$ макс.

3 - повратна тачка

7° ПРЕВОЈНЕ ТАЧКЕ

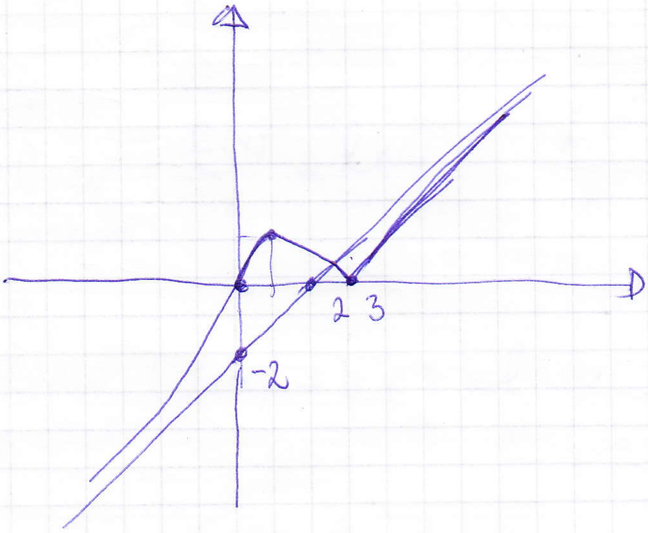
$$f''(x) = \left(\frac{x-1}{\sqrt[3]{x^2(x-3)}}\right)' = \frac{\sqrt[3]{x^2(x-3)} - (x-1)\left(\left(x^2(x-3)\right)^{1/3}\right)'}{\sqrt[3]{x^4(x-3)^2}}$$

$$= \frac{\sqrt[3]{x^2(x-3)} - (x-1)\cdot\frac{1}{3}\left[x^2(x-3)\right]^{-2/3}\left[2x(x-3)+x^2\right]}{x\sqrt[3]{x(x-3)^2}}$$

$$= \frac{3\sqrt[3]{x^2\cdot x^4(x-3)^3} - (x-1)(2x^2-6x+x^2)}{3x\sqrt[3]{x(x-3)^2\cdot x^4\cdot(x-3)^2}} = \frac{3x^2(x-3) - (x-1)(3x^2-6x)}{3x\sqrt[3]{x^5(x-3)^4}}$$

$$= \frac{x^{\cancel{2}}(x-3) - (x-1)\cancel{x}(x-2)}{\cancel{x}\sqrt[3]{x^5(x-3)^4}} = \frac{\cancel{x^2}-3\cancel{x}-(\cancel{x^2}-\cancel{2x}-\cancel{x}+2)}{x\sqrt[3]{x^2(x-3)^4}}$$

$$= \frac{-2}{x(x-3)\sqrt[3]{x^2(x-3)}}$$, $f''(x) \neq 0$ нема превојне тачке



ЗАД2:

$$f(x) = x e^{\frac{1}{x}}$$

1° Д.П. $x \neq 0$, $\forall x \in R \setminus \{0\}$

2° нема нула

3° парност $f(-x) = -x e^{-\frac{1}{x}}$ није ни парна ни непарна

4° знак

$f(x) > 0$, $x > 0$

$f(x) < 0$, $x < 0$

5° асимптоте

В.А. $\lim\limits_{x \nearrow 0} x e^{\frac{1}{x}} = 0$ – нема В.А.

$\lim\limits_{x \downarrow 0} x e^{\frac{1}{x}} = \lim\limits_{x \downarrow 0} \dfrac{e^{\frac{1}{x}}}{\frac{1}{x}}$ ЛП

$= \lim\limits_{x \downarrow 0} \dfrac{e^{\frac{1}{x}} \cdot \cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = +\infty$

<u>КОСА</u>

$$k=\lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{\cancel{x}\,e^{\frac{1}{x}\to 0}}{\cancel{x}}=\lim\;1$$

$$n=\lim_{x\to\pm\infty}(f(x)-kx)=\lim_{x\to\pm\infty}\underbrace{(xe^{\frac{1}{x}}-x)}_{\infty-\infty}$$

$$=\lim_{x\to\pm\infty}\underbrace{x\left(e^{\frac{1}{x}}-1\right)}_{\infty\cdot 0}=\lim_{x\to\pm\infty}\frac{e^{\frac{1}{x}}-1}{\frac{1}{x}}\qquad \text{Л-П.}$$

$$=\lim_{x\to\pm\infty}\frac{e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}=1$$

$$\boxed{y=x+1}$$

6° ЕКСТРЕМИ

$$f'(x)=\left(xe^{\frac{1}{x}}\right)'=e^{\frac{1}{x}}+x\cdot e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)=e^{\frac{1}{x}}\left(1-\frac{1}{x}\right)$$

$$f'(x)=e^{\frac{1}{x}}\,\frac{x-1}{x}$$

$f'(x)=0,\quad x=1\quad,\quad f(1)=e$

$(1,e)$ мин

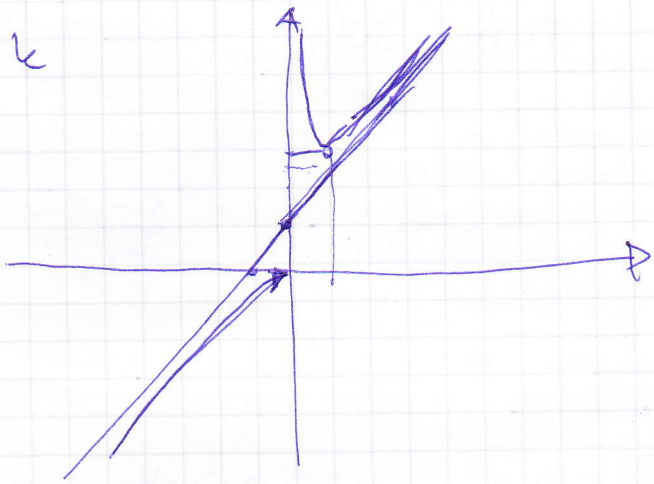
7° ПРЕВОЈНЕ ТАЧКЕ

$$f''(x) = \left(e^{\frac{1}{x}} \frac{x-1}{x}\right)' = e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)\cdot\frac{x-1}{x} + e^{\frac{1}{x}} \frac{x-(x-1)}{x^2}$$

$$= e^{\frac{1}{x}}\left(\frac{1-x}{x^3} + \frac{1}{x^2}\right) = e^{\frac{1}{x}} \frac{1-\cancel{x}+\cancel{x}}{x^3} = \frac{e^{\frac{1}{x}}}{x^3}$$

Нема превојне тачке

8 ГРАФИК



ЗАД 3: $f(x) = \dfrac{-8-x^2}{\sqrt{x^2-4}}$

1° Д.П. $x^2-4>0$, $x \in (-\infty,-2) \cup (2,+\infty)$

2° НУЛЕ, $f(x)=0$, $-8-x^2=0 \Rightarrow x^2=-8$
нема нуле

3° пресек са y-осом.
$x=0$, нема због Д.П.

4° ПАРНОСТ
$f(-x) = \dfrac{-8-x^2}{\sqrt{x^2-4}}$, парна

5° ЗНАК
$\sqrt{x^2-4} > 0$, $\forall x \in$ Д.П. $-8-x^2 < 0$ $\forall x \in$ Д.П
па је $f(x) < 0$ за свако $x \in$ Д.П.

6. Асимптоте.
В.А.
$$\lim_{x \nearrow (-2)} \frac{-8-x^2}{\sqrt{x^2-4}} = -\infty,$$
$$\lim_{x \downarrow (-2)} \frac{-8-x^2}{\sqrt{x^2-4}} = -\infty$$

косa

$$k=\lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{\frac{-8-x^2}{\sqrt{x^2-4}}}{x}=\lim_{x\to\pm\infty}\frac{-8-x^2}{x\sqrt{x^2-4}}$$

$$=-\lim_{x\to\pm\infty}\frac{x^2+8}{x|x|\sqrt{1-\frac{4}{x^2}}}$$

$$k_1=-\lim_{x\to+\infty}\frac{x^2+8}{x^2\sqrt{1-\frac{4}{x^2}}}=-1$$

$$k_2=-\lim_{x\to-\infty}\frac{x^2+8}{-x^2\sqrt{1-\frac{4}{x^2}}}=1$$

$$n_1=\lim_{x\to+\infty}\left(-\frac{8+x^2}{\sqrt{x^2-4}}+x\right)=\lim_{x\to+\infty}\left(x-\frac{8+x^2}{\sqrt{x^2-4}}\right)\cdot\frac{x+\frac{8+x^2}{\sqrt{x^2-4}}}{-1/-}$$

$$=\lim_{x\to+\infty}\frac{x^2-\frac{(8+x^2)^2}{x^2-4}}{x+\frac{8+x^2}{\sqrt{x^2-4}}}=\lim_{x\to+\infty}\frac{\frac{\cancel{x^4}-4x^2-64-16x^2-\cancel{x^4}}{x^2-4}}{x+\frac{8+x^2}{\sqrt{x^2-4}}}$$

$$=\lim_{x\to+\infty}\frac{\frac{-20x^2-64}{x^2-4}\to -20}{\underbrace{x}_{+\infty}+\underbrace{\frac{8+x^2}{\sqrt{x^2-4}}}_{+\infty}}=0$$

аналогно

$$n_2=\lim_{x\to-\infty}\left(-\frac{8+x^2}{\sqrt{x^2-4}}-x\right)=\dots=0$$

(на основу парности функције

$y=-x$, $x\to+\infty$

$y=x$, $x\to-\infty$

Екстреми

$$f'(x)=\frac{-2x\sqrt{x^2-4}+(8+x^2)\frac{2x}{\cancel{2}\sqrt{x^2-4}}}{x^2-4}=\frac{-2x(x^2-4)+x(8+x^2)}{(x^2-4)\sqrt{x^2-4}}$$

$$=\frac{-2x^3+8x+8x+x^3}{(x^2-4)\sqrt{x^2-4}}=\frac{16x-x^3}{(x^2-4)^{3/2}}=\frac{x(16-x^2)}{(x^2-4)^{3/2}}$$

$$f'(x)=0 \Rightarrow \underbrace{x_1=0}_{\notin Д.П.},\quad 16-x^2=0 \Rightarrow x_{2,3}=\pm 4$$

$$f(-4)=\frac{-8-16}{\sqrt{16-4}}=\frac{-20}{2\sqrt{3}}=\frac{-10}{\sqrt{3}}=f(4)$$

~~$f(4)=$~~

знак $f'(x)$, $\quad \dfrac{x(16-x^2)}{(x^2-4)^{3/2}}$, $(x^2-4)^{3/2}>0$ због Д.П.

	$-\infty$ … -4	-4 … -2	2 … 4	4 … $+\infty$
x	$-$	$-$	$+$	$+$
$4-x$	$+$	$+$	$+$	$-$
$4+x$	$-$	$+$	$+$	$+$
	$+$	$-$	$+$	$-$

$f'(x)>0$, $x\in(-\infty,-4)\cup(2,4)$

$f'(x)<0$, $x\in(-4,-2)\cup(4,+\infty)$

$\left(\pm 4, -\frac{10}{\sqrt{3}}\right)$ има макс.

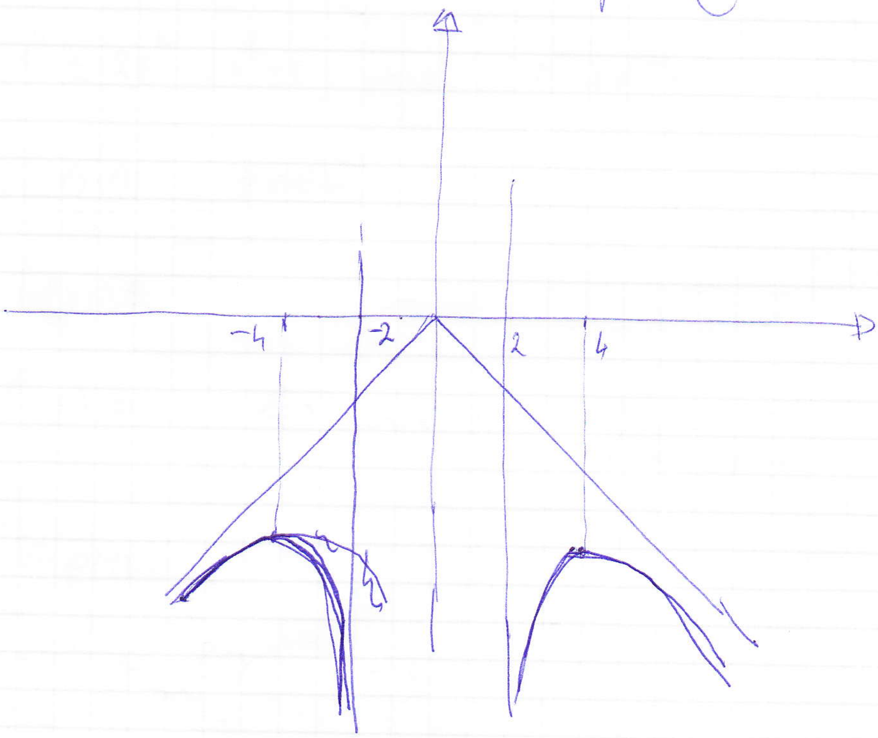
ПРЕВОЈНЕ ТАЧКЕ:

$$f''(x) = \frac{(16-3x^2)(x^2-4)^{3/2} - (16x-x^3)\cdot\frac{3}{\cancel{2}}(x^2-4)^{\frac{1}{2}}(\cancel{2}x)}{(x^2-4)^3}$$

$$= \frac{(x^2-4)^{1/2}\left[(16-3x^2)\cdot(x^2-4) - (16x-x^3)\cdot 3x\right]}{(x^2-4)^3}$$

$$= \frac{16x^2 - 64 - \cancel{3x^4} + 12x^2 - 48x^2 + \cancel{3x^4}}{(x^2-4)^{5/2}} = \frac{-20x^2-64}{(x^2-4)^{5/2}} = \frac{-4(5x^2+16)}{(x^2-4)^{5/2}}$$

$f''(x) \neq 0$ – нема превојне тачке



(И.З.) **ЗАД 4.** Испитати и графички представити функцију

$$f(x) = \arcsin \frac{x^2}{\sqrt{2x^4-2x^2+1}}$$

Решење:

1° Д.П.

$$-1 \leq \frac{x^2}{\sqrt{2x^4-2x^2+1}} \leq 1 \quad \Big/ \cdot \sqrt{2x^4-2x^2+1} > 0 \text{ за } \forall x \in R$$

$$-\sqrt{2x^4-2x^2+1} \leq x^2 \quad T \quad и$$

$$x^2 \leq \sqrt{2x^4-2x^2+1} \quad \Big/^2$$

$$x^4 \leq 2x^4-2x^2+1 \Rightarrow x^4-2x^2+1 \geq 0 \Rightarrow (x^2-1)^2 \geq 0 \quad T$$

тј. Д.П. $\forall x \in R$

2° Нуле

$$f(x)=0 \Leftrightarrow \arcsin \frac{x^2}{\sqrt{2x^4-2x^2+1}} = 0 \Leftrightarrow \boxed{x=0}$$

3° Парност

$f(-x) = f(x)$ − парна

4° Знак

$f(x) > 0$, $\forall x \in R$ јер је $\frac{x^2}{\sqrt{2x^4-2x^2+1}} > 0$

$$\arcsin \frac{x^2}{\sqrt{2x^4-2x^2+1}} > 0$$

јер arcsin − је растућа функ.

5° асимптоти

нема В.А.

Коса

$$k=\lim_{x\to\pm\infty}\frac{\arcsin\frac{x^2}{\sqrt{2x^4-2x^2+1}}}{x}=0 \qquad \frac{1}{\sqrt2}\quad \frac{\pi}{4}$$

нема К.А.

Х.А.

$$\lim_{x\to\pm\infty}\arcsin\frac{x^2}{\sqrt{2x^4-2x^2+1}}=\frac{\pi}{4}$$

6° екстреми

$$f'(x)=\frac{1}{\sqrt{1-\frac{x^4}{2x^4-2x^2+1}}}\cdot\frac{2x\sqrt{2x^4-2x^2+1}-x^2\cdot\frac{\overset{4}{\cancel{8}}x^3-\overset{2}{\cancel{4}}x}{\cancel{2}\sqrt{2x^4-2x^2+1}}}{2x^4-2x^2+1}$$

$$=\frac{\cancel{\sqrt{2x^4-2x^2+1}}}{\sqrt{x^4-2x^2+1}}\cdot\frac{2x(2x^4-2x^2+1)-x^2(4x^3-2x)}{(2x^4-2x^2+1)\cdot\cancel{\sqrt{2x^4-2x^2+1}}}$$

$$=\frac{\cancel{4x^5}-4x^3+2x-\cancel{4x^5}+2x^3}{\sqrt{(x^2-1)^2}\,(2x^4-2x^2+1)}=\frac{-2x^3+2x}{|x^2-1|(2x^4-2x^2+1)}=\frac{2x(1-x^2)}{|x^2-1|(2x^4-2x^2+1)}$$

$$f'(x)=\begin{cases}\dfrac{-2x}{\underbrace{2x^4-2x^2+1}_{>0}}, & x\in(-\infty,-1)\cup(1,+\infty)\\[2ex] \dfrac{2x}{\underbrace{2x^4-2x^2+1}_{>0}}, & x\in(-1,1)\end{cases}$$

	$-\infty$		-1		0		1		$+\infty$
f'		+	Н.В	−		+	Н.В.	−	
f		↗	max	↘	min	↗	max	↘	

$f(\pm 1) = \arcsin 1 = \frac{\pi}{2}$

7° превојне тачке

$$f''(x) = \begin{cases} \frac{2(6x^4-2x^2-1)}{(2x^4-2x^2+1)^2}, & x \in (-\infty,-1) \cup (1,+\infty) \\ -\frac{2(6x^4-2x^2-1)}{(2x^4-2x^2+1)^2}, & x \in (-1,1) \end{cases}$$

$f''(x) = 0 \iff 6x^4 - 2x^2 - 1 = 0$

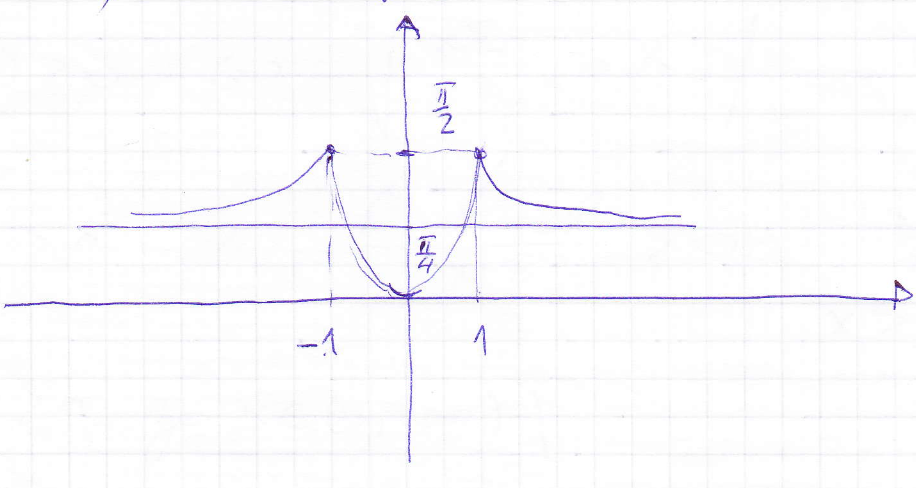
смјена: $x^2 = t$, $6t^2 - 2t - 1 = 0$

$$t_{1,2} = \frac{2 \pm \sqrt{4+24}}{12} = \frac{2 \pm 2\sqrt{7}}{12} = \frac{1 \pm \sqrt{7}}{6}$$

$x^2 = \frac{1 \pm \sqrt{7}}{6}$, $x_{1,2} = \pm\sqrt{\frac{1+\sqrt{7}}{6}}$

$x_{3,4}$ – комплексна рјешења

$x_1 \approx 0{,}78$, $x_2 \approx -0{,}78$



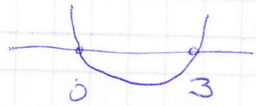
$\boxed{\text{зад } 5}$ Испитати и графички представити функцију

$$f(x) = -x + \left[x^3(x-3)^{-1}\right]^{1/2}$$

Решење:

$$f(x) = -x + \sqrt{\frac{x^3}{x-3}}$$

1° д.п. $\frac{x^3}{x-3} \geq 0$ тј. $\frac{x}{x-3} > 0$



$x \in (-\infty, 0] \cup (3, +\infty)$

2° нуле, $f(x) = 0 \iff -x + \sqrt{\frac{x^3}{x-3}} = 0$

$\sqrt{\frac{x^3}{x-3}} = x \quad /^2 \qquad x \geq 0$

$x^3 = x^2(x-3) \Rightarrow \cancel{x^3} = \cancel{x^3} - 3x^2 \Rightarrow \boxed{x = 0}$

3° парност

$f(-x) = x + \sqrt{\frac{(-x)^3}{x-3}} = x + \sqrt{\frac{-x^3}{x-3}}$ ни парна ни непарна

4° знак

$f(x) > 0 \iff \sqrt{\frac{x^3}{x-3}} > x \quad /^2 \qquad (x \geq 0)$

$\frac{x^3}{x-3} > x^2 \Rightarrow \frac{\cancel{x^3} - \cancel{x^3} + 3x^2}{x-3} > 0 \qquad \frac{3x^2}{x-3} > 0$

$x > 3$ тј. $x \in (3, +\infty)$

$\boxed{f(x) > 0 \text{ за } x \in (3, +\infty)}$

$f(x)<0$, $\sqrt{\frac{x^3}{x-3}} < x \; \Big/^2$ $(x\geq 0)$

$\frac{3x^2}{x-3} < 0 \Rightarrow x<3$

$f(x)<0$ за $x(-\infty,0)\cap x>0$

тј. функција је позитивна $\forall x \in$ Д.П.

5° асимптоте

В.А. $\lim\limits_{x\downarrow 3}\left(-x+\sqrt{\frac{x^3}{x-3}}\right)=+\infty$

$-3+\infty$

К.А. $k=\lim\limits_{x\to\pm\infty}\frac{-x\pm|x|\sqrt{\frac{x}{x-3}}}{x}$

$k_1=\lim\limits_{x\to+\infty}\frac{-x+x\sqrt{\frac{x}{x-3}}}{x}=\lim\limits_{x\to+\infty}\left(\sqrt{\frac{x}{x-3}}-1\right)=0$ $\to 1$

тј. нема косу асимптоту $x\to+\infty$

$k_2=\lim\limits_{x\to-\infty}\frac{-x-x\sqrt{\frac{x}{x-3}}}{x}=\lim\limits_{x\to-\infty}\left(-\sqrt{\frac{x}{x-3}}-1\right)=-2$

$n_2=\lim\limits_{x\to-\infty}\left(-x-x\sqrt{\frac{x}{x-3}}+2x\right)=\lim\limits_{x\to-\infty}\left(x-x\sqrt{\frac{x}{x-3}}\right)$ $\infty-\infty$

$=\lim\limits_{x\to-\infty}\frac{1-\sqrt{\frac{x}{x-3}}}{\frac{1}{x}}=\lim\limits_{x\to(-\infty)}\frac{-\frac{1}{2}\left(\frac{x}{x-3}\right)^{-1/2}\cdot\frac{x-3-x}{(x-3)^2}}{-\frac{1}{x^2}}=-\frac{3}{2}$ $\to 3$

$y=-2x-\frac{3}{2}$ $(x\to-\infty)$

Х. А.

$$n_1 = \lim_{x \to +\infty} \left(-x + x\sqrt{\frac{x}{x-3}}\right) = \dots = \frac{3}{2}$$

6° ЕКСТРЕМИ

$$f'(x) = -1 + \frac{1}{2}\cdot\left(\frac{x^3}{x-3}\right)^{-1/2} \frac{3x^2(x-3) - x^3}{(x-3)^2}$$

$$f'(x) = -1 + \frac{1}{2}\cdot\left(\frac{x-3}{x^3}\right)^{1/2} \frac{2x^3-9x^2}{(x-3)^2}$$

$$f'(x) = 0 \Rightarrow \frac{1}{2}\sqrt{\frac{x-3}{x^3}}\cdot\frac{2x^3-9x^2}{(x-3)^2} = 1$$

$$\sqrt{\frac{x-3}{x^3}} = \frac{2(x-3)^2}{x^2(2x-9)} \Big/^2 \quad \text{за} \quad 2x-9>0 \Rightarrow \boxed{x > \frac{9}{2}}$$

$x > 4{,}5$

$\boxed{x > 4{,}5}$

$$\frac{\cancel{x-3}}{\cancel{x^3}} = \frac{4(x-3)^{\cancel{4}\,3}}{x^4(2x-9)^2}$$

$$x(2x-9)^2 = 4(x-3)^3$$

$$x(4x^2-36x+81) = 4(x^3-9x^2+27x-27)$$

$$\cancel{4x^3}-\cancel{36x^2}+81x = \cancel{4x^3}-\cancel{36x^2}+108x-108$$

$$27x-108=0 \Rightarrow x = \frac{108}{27} = 4 \quad , \boxed{x=4}$$

Нема екстрема

7° ПРЕВОЈНЕ ТАЧКЕ

$$f''(x) = \frac{1}{4}\left(\frac{x-3}{x^3}\right)^{-1/2}\cdot\frac{x^{\cancel{3}}-(x-3)\cdot3x^{\cancel{2}}}{x^{\cancel{6}\,4}}\cdot\frac{2x^3-9x^2}{(x-3)^2}$$

$$+\frac{1}{2}\left(\frac{x-3}{x^3}\right)^{1/2}\cdot\frac{(6x^2-18x)(x-3)^{\cancel{2}}-(2x^3-9x^2)\cdot2\cancel{(x-3)}}{(x-3)^{\cancel{4}\,3}}$$

$$= \frac{1}{4}\left(\frac{x-3}{x^3}\right)^{-1/2}\cdot\frac{x-3x+9}{x^{\cancel{4}\,2}}\cdot\frac{\cancel{x^2}(2x-9)}{(x-3)^2} + \frac{1}{2}\left(\frac{x-3}{x^3}\right)^{1/2}\frac{6x^3-\cancel{18x^2}-18x^2+54x-4x^3+\cancel{18x^2}}{(x-3)^3}$$

$$f''(x)=\frac{1}{4}\left(\frac{x-3}{x^3}\right)^{-1/2}\frac{-2x+9}{x^2}\cdot\frac{2x-9}{\cancel{x}-3}+\frac{1}{2}\left(\frac{x-3}{x^3}\right)^{1/2}\frac{2x^3-18x^2+54x}{(x-3)^3}$$

$$f''(x)=-\frac{1}{4}\left(\frac{x-3}{x^3}\right)^{-1/2}\frac{(2x-9)^2}{x^2(x-3)}+\frac{1}{2}\left(\frac{x-3}{x^3}\right)^{1/2}\frac{2x(x^2-9x+27)}{(x-3)^3}$$

⋮

$$\boxed{f''(x)=\frac{(9-2x)(27-x^2)}{2x^3(x-3)^2}\sqrt{\frac{x^3}{x-3}}}$$

??

– нема рівностей нуля